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- Page 109, line 12: $4\min(\lambda_j, \lambda_k)\ (\geq 4/N) \implies 2\min(\lambda_j, \lambda_k)\ (\geq 2/N)$
- Page 113, line 12 from bottom (last paragraph of the proof of Theorem 4.18): Let p be such that the optimal solutions to (A) with respect to p form a minimal face of the feasible region of (A).
- Page 133, (6.2): $\Delta f(z, v, u) \implies \Delta f(z; v, u)$
- Page 139, Proposition 6.8 (2): Condition (6.26) follows from (6.27), and hence (6.26) is redundant.
- Page 143, Theorem 6.13 (8) [Convolution of M-convex functions]
  The proof here makes use of transformation by a network, but an alternative direct proof can be found in:
  K. Murota: On infimal convolution of M-convex functions, RIMS Kokyuroku, No.1371 (2004), 20–26, and METR 2004-12, Department of Mathematical Informatics, University of Tokyo, March 2004
  http://www.keisu.t.u-tokyo.ac.jp/research/techrep/data/2004/METR04-12.pdf
- Page 151, Theorem 6.30, Proof of Claim 1:

  The final step reads: "This shows (B-EXC$_+$[$\mathbf{R}$]) for $\overline{B}$. Therefore, B is an M-convex set." Before we can argue in this way, we have to verify $\overline{B} \cap \mathbf{Z}^V = B$, which is possible.
- Page 152, Section 6.8: A characterization of gross substitutes property in terms of an exchange property is also found in:
  H. Reijnierse, A. van Gallekom, and J. A. M. Potters: Verifying gross substitutability, *Economic Theory*, **20** (2002), 767–776.
- Page 172, Proof of Theorem 6.74:

  "Theorem 6.4 can be strengthened to a statement that (M-EXC[$\mathbf{Z}$]) and (M-EXC$_{\rm loc}$[$\mathbf{Z}$]) are equivalent if $\mathrm{dom}\, f$ satisfies (Q-EXC$_{\rm w}$). (This can be shown by modifying the proof of Claim 2 in the proof of Theorem 6.4.)"

  The detail of the argument can be found in a memorandum of A. Shioura: Level set characterization of M-convex functions (February 1998); see Claim 2 on page 6.
- Page 185, Theorem 7.14 [L-optimality criterion]
  The proof here makes use of the optimality criterion for integrally convex functions, but an alternative direct proof can be found in:

K: Murota: A proof of the L-optimality criterion theorem, unpublished note, July 2004,
http://www.misojiro.t.u-tokyo.ac.jp/ murota/paper/loptimality04.pdf

- Page 219, Theorem 8.17 [M-convex intersection theorem]
  The proof here makes use of the M-separation theorem, but an alternative direct proof can be found in:
  K. Murota: A proof of the M-convex intersection theorem, RIMS Kokyuroku, No.1371 (2004), 13–19, and METR 2004-03, Department of Mathematical Informatics, University of Tokyo, January 2004
  http://www.keisu.t.u-tokyo.ac.jp/research/techrep/data/2004/METR04-03.pdf

- Page 305, Section 10.3.1:

  A detailed analysis of the steepest descent algorithm for L-convex functions can be found in:

  K. Murota and A. Shioura: Exact bounds for steepest descent algorithms of L-convex function minimization, *Operations Research Letters*, **42** (2014), 361–366.

- Page 365, [39]: P. G. Doyle and J. L. Snell: *Random Walks and Electrical Networks*, Mathematical Association of America, Washington DC, 1984.

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